

Introduction to Nonlinear Dynamics: Final exam

November 25, 2005

Attempt any five questions. Each question is worth 20 points.

1. Find all solutions to the dynamical system.

$$\dot{x} = -7x + y \quad (1)$$

$$\dot{y} = -7y \quad (2)$$

2. For which of the following matrices A is the dynamical system $\dot{U} = AU$ hyperbolic?

(a) $\begin{pmatrix} -4 & 5 \\ 6 & 4 \end{pmatrix}$

(b) $\begin{pmatrix} -4 & 5 \\ 8 & -10 \end{pmatrix}$

3. Consider the dynamical system

$$\dot{x} = y \quad (3)$$

$$\dot{y} = -x \quad (4)$$

What are the closed orbits of this system?

4. Consider a smooth planar dynamical system. Show that if the orbit through some point in the plane is bounded, then some point in the plane is an equilibrium point.
5. Show that if $f : [0, 1] \rightarrow [0, 1]$ is a homeomorphism fixing the endpoints of the interval, then there is a homeomorphism $g : [0, 1] \rightarrow [0, 1]$ such that $f = g \circ g$.
6. Show that there is an orientation preserving homeomorphism $f : S^1 \rightarrow S^1$ without square roots, i.e., such that there does not exist an orientation preserving homeomorphism $g : S^1 \rightarrow S^1$ with $f = g \circ g$.